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Realistic anomaly-mediated supersymmetry breaking

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Abstract: We consider supersymmetry breaking communicated entirely by the superconformal anomaly in supergravity. This scenario is naturally realized if supersymmetry is broken in a hidden sector whose couplings to the observable sector are suppressed by *more* than powers of the Planck scale, as occurs if supersymmetry is broken in a parallel universe living in extra dimensions. This scenario is extremely predictive: soft supersymmetry breaking couplings are completely determined by anomalous dimensions in the effective theory at the weak scale. Gaugino and scalar masses are naturally of the same order, and flavor-changing neutral currents are automatically suppressed. The most glaring problem with this scenario is that slepton masses are negative in the minimal supersymmetric standard model. We point out that this problem can be simply solved by coupling extra Higgs doublets to the leptons. Lepton flavor-changing neutral currents can be naturally avoided by approximate symmetries. We also describe more speculative solutions involving compositeness near the weak scale. We then turn to electroweak symmetry breaking. Adding an explicit μ term gives a value for $B\mu$ that is too large by a factor of ~ 100. We construct a realistic model in which the μ term arises from the vacuum expectation value of a singlet field, so *all* weak-scale masses are directly related to $m_{3/2}$. We show that fully realistic electroweak symmetry breaking can occur in this model with moderate fine-tuning.

Keywords: Supersymmetry Breaking, Supergravity Models, Beyond Standard Model, Supersymmetric Standard Model.

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Contents

1. Introduction

The hidden-sector scenario for supersymmetry (SUSY) breaking is arguably the simplest and most natural mechanism for realizing SUSY in nature [1]. In this scenario, one assumes that SUSY is broken in a hidden sector that couples only gravitationally to observable fields. Because supergravity (SUGRA) couples universally to all fields, it necessarily connects the observable and hidden sectors, and therefore communicates SUSY breaking to the observable sector. This scenario is very attractive from a theoretical point of view because all of the ingredients are either there of necessity (e.g. SUGRA) or arise naturally (e.g. hidden sectors are a generic consequence of string theory).

SUGRA interactions are flavor-blind, so one might hope that this scenario will not give rise to off-diagonal terms in the squark masses that can lead to flavor-changing neutral currents. Unfortunately, it is very difficult to suppress higher-dimension operators of the form

$$\mathcal{L}_{\text{eff}} \sim \int d^4\theta \, \frac{1}{m_*^2} X^\dagger X Q^\dagger Q \,, \tag{1.1}$$

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where X is a field in the hidden sector, Q is a field in the observable sector, and $m_* \sim 2 \times 10^{18}$ GeV is the reduced Planck mass. If SUSY is broken by $\langle F_X \rangle \neq 0$, this will give rise to soft masses $m_{\tilde{Q}} \sim \langle F_X \rangle / m_* \sim m_{3/2}$. The difficulty is that there is *a priori* no reason that the terms eq. (1.1) (and hence the scalar masses) should be flavor-diagonal, and this will lead to unacceptable flavor-changing neutral currents (FCNC's) unless [2]

$$\frac{m^2_{\tilde{d}\tilde{s}}}{m^2_{\tilde{s}}} \lesssim (5 \times 10^{-3}) \left(\frac{m_{\tilde{s}}}{1\,\mathrm{TeV}} \right), \qquad \mathrm{Im} \left(\frac{m^2_{\tilde{d}\tilde{s}}}{m^2_{\tilde{s}}} \right) \lesssim (5 \times 10^{-4}) \left(\frac{m_{\tilde{s}}}{1\,\mathrm{TeV}} \right). \tag{1.2}$$

We know that SUGRA is an effective theory with a cutoff of order m_*, and we expect that the fundamental theory above this cutoff does not conserve flavor.[1] The operator eq. (1.1) is therefore allowed by all symmetries, and there seems to be no natural way to suppress it. This is the famous SUSY flavor problem.

An elegant solution to this problem was proposed by Randall and Sundrum in ref. [4]. They considered a scenario in which there are one or more compact extra dimensions with a size $R \gg 1/M_*$, where M_* is the fundamental (higher-dimensional) Planck scale. They further assumed that the observable sector and SUSY breaking sector are localized on separate $(3+1)$-dimensional subspaces in the extra dimensions. These subspaces may arise as topological defects in field theory, or D-branes or orbifold fixed points in string theory; their precise nature is not essential for the present purpose, and we will refer to them generically as "3-branes". For simplicity, we assume that the separation between the branes in the extra dimensions is also of order R. We also assume that the only fields with mass below M_* that propagate in the higher dimensions between the branes are the SUGRA fields (which are necessarily present, since gravity is the dynamics of spacetime itself).

Scenarios with SUSY breaking localized on orbifold fixed points have been considered previously by Hořava and Witten in the context of M-theory [5]. There the existence of a large compactification radius reduces the fundamental string scale and allows gauge-gravity unification. String vacua with gauge and/or matter fields localized on D-branes have been considered by several authors [6]. The observation of ref. [4] that the SUSY flavor problem can be solved in an elegant way in models of this type gives a strong additional motivation for these scenarios.

The key observation of ref. [4] is that in this scenario operators such as eq. (1.1) are not present in the effective theory below M_* for the excellent reason that they are not local operators! Even if we assume that the theory above the scale M_* violates flavor maximally, the only flavor-violating couplings between the observable and hidden sectors comes from the exchange of quanta with Planck-scale masses

[1] One can consider models in which the flavor symmetry is a gauged symmetry at the Planck scale. However, flavor symmetry must be broken to obtain the observed quark and lepton masses, and it is non trivial to do this while maintaining sufficient degeneracy among squarks to avoid FCNC's [3].

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between the branes. But these effects are exponentially suppressed by $e^{-M_* R}$ due to the spatial fall-off of massive propagators. The leading coupling between the hidden and observable sectors comes from SUGRA fields, whose couplings are flavor-blind. Note also that due to the exponential suppression of flavor-changing effects, this scenario requires only a very modest hierarchy between R and M_*.

Below energy scales of order $1/R$, the effective theory becomes $(3+1)$-dimensional, with the hidden and observable sectors inhabiting the same space. This effective lagrangian contains higher-dimension operators connecting the hidden and observable sectors from integrating out Kaluza-Klein modes, but these effects conserve flavor and therefore cannot contribute to FCNC's. Furthermore, these effects do not compete with anomaly mediation unless $R \sim 1/M_*$, as discussed below.

If we assume that the observable sector contains only renormalizable couplings, tree-level SUGRA effects do not give rise to soft SUSY breaking in the observable sector. At loop level, soft SUSY breaking is generated, in a way that is connected in a precise way to the conformal anomaly [4, 7]. This leads to an extremely predictive scenario: *all* of the soft SUSY breaking parameters are determined by $m_{3/2}$ and anomalous dimensions at the electroweak scale. Specifically,

$$m_\lambda = \frac{\beta(g^2)}{2g^2} m_{3/2}\,, \qquad m^2_{\tilde{Q}} = -\frac{1}{4}\frac{d\gamma_Q}{d\ln\mu} m^2_{3/2}\,, \tag{1.3}$$

where

$$\beta(g^2) \equiv \frac{dg^2}{d\ln\mu}\,, \qquad \gamma_Q \equiv \frac{d\ln Z_Q}{d\ln\mu} \tag{1.4}$$

are the gauge beta function and matter field anomalous dimensions. Equations (1.3) are *exact* formulas for the SUSY breaking parameters in a superfield coupling scheme [8, 7] if we ignore quantum gravity corrections. An important feature of these formulas is that scalar and gaugino masses are of the same order

$$m_\lambda \sim m_{\tilde{Q}} \sim \mathrm{M_{SUSY}} = \frac{m_{3/2}}{16\,\pi^2}\,. \tag{1.5}$$

These results hold in any scenario with additional suppressions between the hidden and observable sectors. We therefore refer to this scenario as "anomaly-mediated SUSY breaking".

The quark anomalous dimensions (and hence the squark masses) are dominated by the contribution of $\mathrm{SU(3)_C}$, which is flavor-independent. The only flavor-dependent contributions come from the quark Yukawa couplings, which are small for the first two generations, so FCNC's are suppressed.[2]

Anomaly mediation is clearly an attractive scenario, but in its simplest form, it is not realistic. The most glaring problem is that if the observable sector consists of the minimal supersymmetric standard model (MSSM), all slepton mass-squared terms

[2]A detailed study of FCNC's for the third generation in this scenario would be worthwhile.

are negative, leading to a spontaneous breaking of electromagnetism. Reference [4] considered the possibility that this is cured by having additional interactions in the bulk coupling the leptons to the hidden sector. However, these new contributions depend on additional parameters that must be adjusted to special values in order to obtain soft masses of order eq. (1.5). Therefore, in such scenarios the fundamental requirement that all soft masses are of the same order does not arise naturally. In this paper, we explore the alternative that all SUSY breaking arises from anomaly mediation, so that eq. (1.5) is automatic. This requires an extension of the MSSM. In order to solve the problem of slepton masses, at least some of the fields beyond the MSSM must have masses near the weak scale. This is because the slepton masses are determined by the anomalous dimensions at the weak scale. As a result, this kind of model is directly testable in accelerator experiments, giving an additional motivation to study such models.

We consider several possibilities for obtaining positive slepton mass-squared terms. One simple possibility is to add an extra pair of Higgs doublets with large (order 1) Yukawa couplings to leptons. Lepton FCNC's can be naturally suppressed by approximate flavor symmetries if we introduce 3 extra pairs of Higgs doublets. In order to preserve one-step gauge unification, one can contemplate adding 3 color triplets, so that the extra fields form complete $(\mathbf{5}\oplus\bar{\mathbf{5}})$'s of SU(5). However, this makes the $\mathrm{SU}(3)_\mathrm{C}$ beta function vanish at one loop, leading to squark masses that are too small. We therefore give up one-step gauge coupling in this approach, although of course models of this type may be embedded in a non-minimal grand unified theory, e.g. with intermediate scales.

We also briefly discuss a more speculative mechanism for positive slepton masses involving compositeness at the weak scale.

We then turn to electroweak symmetry breaking. Introducing a tree-level μ term spoils the relation eq. (1.5), which is required for a realistic model. This is a direct reult of introducing a dimensionful parameter into the theory. (Specifically, $B\mu = m_{3/2}\mu \sim 16\,\pi^2 \mu\, M_\mathrm{SUSY}$). The Giudice-Masiero mechanism [9] for generating a μ term is not available if we do not want to introduce additional couplings between the hidden and observable sectors.[3] The remaining possibility is that an effective μ term is generated by the vacuum expectation value (VEV) of a singlet at the weak scale. Motivated by the solution of the slepton mass problem, we consider a model with 3 extra Higgs doublets, 1 vector-like pair of color triplets, and 4 singlets. The color triplets are needed in order to obtain a negative mass-squared for the singlet whose VEV gives the μ term. The other singlets give important contributions to the soft terms for the ordinary Higgs fields required for electroweak symmetry breaking. The model contains no dimensionful parameters, so *all* mass scales are set by anomaly

[3] Even if we do introduce such interactions, special parameter choices are required to obtain $B\mu \sim \mu\, M_\mathrm{SUSY}$ [4].

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mediation: in this sense, this model takes the idea of radiative symmetry breaking to its logical extreme. It is remarkable that the masses of all superpartners can be given phenomenologically acceptable values through this mechanism, with only moderate fine-tuning ($\sim 1/20$).

This paper is organized as follows. In section 2, we review SUSY breaking on a parallel universe and anomaly mediation. In section 3, we present our solutions to the slepton mass problem. In section 4, we show how electroweak symmetry breaking (including the μ term) can arise entirely from anomaly mediation. section 5 contains our conclusions.

2. Review of anomaly-mediated supersymmetry breaking

In this section, we give a brief review of anomaly-mediated SUSY breaking. No originality is claimed here, but we hope that an overview will be useful to the reader. We also wish to emphasize the simple "1PI" understanding of anomaly mediation described in ref. [7].

2.1 Parallel universes

Anomaly-mediated SUSY breaking is the leading effect in models where higher-dimension operators connecting the hidden and observable sectors have coefficients that are small in units of the Planck mass. Reference [4] pointed out that this scenario is naturally obtained if the hidden sector is localized on a parallel universe in extra dimensions. Although anomaly-mediation is in principle more general than this scenario, we briefly review some of the most important features of SUSY breaking communicated from a spatially separated "3-brane" to see how some of the general features discussed below can arise in a specific context.

Our starting point is the assumption that there are $n \geq 1$ extra dimensions compactified with a radius $R \gg 1/M_*$, where M_* is the $(4+n)$-dimensional Planck scale. When we perform numerical estimates we will want to include e.g. factors of π, and for simplicity we will take the compactified space to be a symmetric torus with radius R. Furthermore, we assume that the standard-model fields are localized on a $(3+1)$-dimensional subspace ("3-brane"), and the hidden sector that breaks SUSY is localized on a 3-brane spatially separated from the observable 3-brane by a distance $\sim \pi R$. (This is the maximum distance between two points on a circle of radius R, and the most natural choice for the separation of the 3-branes.) We also assume that the only light (below M_*) fields propagating between the branes are SUGRA fields. This is a very strong assumption, and we will briefly consider the extent to which it can be relaxed below. We will not address the question of how such a scenario can arise from Planck-scale physics, but we note that extra dimensions and branes (in our generalized sense) with localized degrees of freedom are generic features of string

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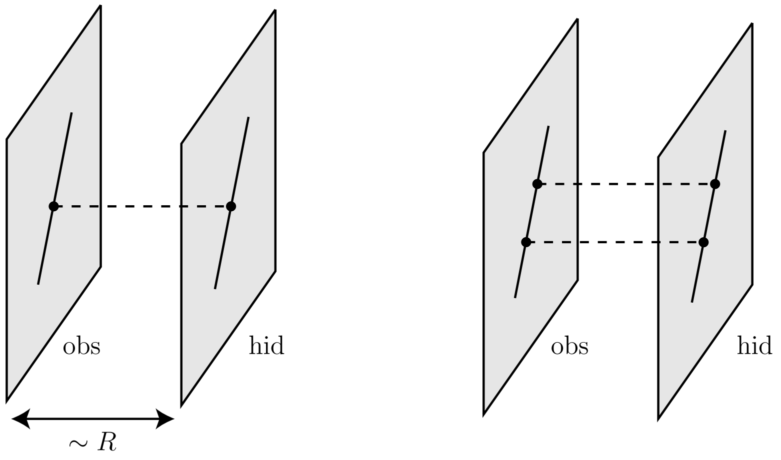

(1a) (1b)

Figure 1: Diagrams that contribute to quartic terms in the Kähler potential connecting the hidden and observable sectors. The solid lines correspond to fields localized on the 3-branes, while the dashed lines correspond to SUGRA fields propagating in the bulk.

vacua. We will simply assume that such a configuration exists (and is stable) and work out the consequences.

We do this by writing an effective theory below the scale M_* that includes the branes, the fields localized on them, and SUGRA. The effective action for such a theory is

$$S_{\text{eff}}^{(4+n)} = \int d^4x\, d^ny \left\{ \delta^n(y - y_{\text{obs}})\mathcal{L}_{\text{obs}}^{(4)} + \delta^n(y - y_{\text{h}id})\mathcal{L}_{\text{hid}}^{(4)} + \mathcal{L}_{\text{bulk}}^{(4+n)} \right\}, \tag{2.1}$$

where y are the coordinates corresponding to the extra dimensions. (We are considering the simple case where the hidden and observable sectors are localized at fixed values of y.) We do not make any assumptions about the symmetry structure of the theory above the scale M_*, and so we include all higher-dimension operators consistent with gauge symmetries. However, there are no higher-dimension operators connecting the fields in the hidden and observable sectors because such operators are not local. In fact, if we integrate out heavy modes with masses of order M_* that propagate between the hidden and observable sectors, these will give effects suppressed by $e^{-\pi R M_*}$ because of the exponential decay of a massive propagator in position space (in any dimension).[4] See figure $1a$. These effects are exponentially suppressed if

[4]As discussed above, these exponentially suppressed effects cannot be parameterized by local terms in the effective field theory below M_*. This would seem to imply that there is no way to consistently include these effects in an effective field theory description. We believe that the resolution to this apparent paradox is that effective field theory is capable of reproducing the results of the full theory *in an expansion in* $1/M_*$. The exponential effects are smaller than any power of $1/M_*$, and hence fall outside the domain of effective field theory.

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$R \gg 1/M_*$; in fact, we need only a modest hierarchy (e.g. $M_* R \sim 3$) in order to be completely safe from FCNC's induced by higher-dimension operators.[5] The fact that very modest hierarchies of scales are required is one of the most appealing features of this scenario.

We can now construct the effective theory below the scale R^{-1}. This can be viewed as integrating out all the Kaluza-Klein (KK) excitations of the bulk SUGRA modes with mass $M_{\rm KK} \sim R^{-1}$. The resulting theory is a 4-dimensional effective SUGRA theory with a cutoff $\Lambda \sim M_{\rm KK}$ and a 4-dimensional Planck scale given by

$$m_*^2 = V_n M_*^{2+n}\,, \tag{2.2}$$

where V_n is the volume of the extra dimensions ($V_n = (2\pi R)^n$ for a symmetric torus). An important point is that this matching involves integrating out only SUGRA modes, and therefore does not induce flavor-dependent operators. However, we must estimate these matching effects to see whether they give significant contributions to flavor-diagonal soft terms. Exchange of a single SUGRA KK mode (as in figure 1a) is suppressed by $1/(m_*^2 M_{\rm KK}^2)$, with the 4-dimensional Planck suppression arising from the gravitation coupling and $M_{\rm KK}$ from the massive propagator. These diagrams therefore give higher-derivative operators such as

$$\Delta\mathcal{L}^{(4)} \sim \frac{1}{m_*^2 M_{\rm KK}^2} \int d^4\theta\, X^\dagger X Q^\dagger \Box\, Q \tag{2.3}$$

that do not contribute to soft masses. There are also loop effects connecting the hidden and observable sectors, as in figure 1b. If the quanta being exchanged are massless SUGRA modes, this diagram gives a 1-loop matching correction to the 4-dimensional effective theory, which gives an operator of the form

$$\Delta\mathcal{L}^{(4)} \sim \frac{1}{16\,\pi^2}\frac{R^{-2}}{m_*^4} \int d^4\theta\, X^\dagger X Q^\dagger Q\,. \tag{2.4}$$

(The appearance of the 4-dimensional Planck scale can be understood directly by evaluating this graph in the 4-dimensional effective theory, or by keeping track of the volume factors in the $(4+n)$-dimensional graph.) These operators can compete with the anomaly-mediated contributions if the hierarchy between M_* and R is modest. For example, for n extra dimensions compactified on a symmetric torus of radius R, the contributions above will dominate the anomaly-mediated contributions (discussed below) provided

$$M_* R \lesssim \left(\frac{16\pi^2}{(2\pi)^n}\right)^{1/(2+n)}. \tag{2.5}$$

This goes to zero for large n, but e.g. for $n = 1$, this is $M_* R \lesssim 3$, while the exponential suppression of states with mass M_* propagating between the hidden and

[5]This does not forbid operators that violate e.g. baryon number within the visible sector. However, such operators can be suppressed in other ways, such as imposing extra gauge symmetries.

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observable sector is $e^{\pi R M_*} \sim 10^{-4}$ (assuming the separation between the sectors is maximal). Although there are large uncertainties in these estimates, they do show that models of this type are plausible. This was invoked in ref. [4] as a possible mechanism to cure the problem with negative slepton masses.[6] In this paper we will investigate the more robust possibility that $M_* R$ is large enough that these effects can be neglected.

From the above discussion, it is clear that one can consider more general scenarios with additional bulk fields, as long as these fields do not have flavor-dependent couplings. It may be difficult to eliminate dangerous flavor-changing bulk fields in realistic string models. Our focus is on the phenomenology of anomaly-mediation, and we will not attempt to address this question here.

With these results we can write the 4-dimensional effective theory below the scale $1/R$. Neglecting exponentially small effects and operators of the form eq. (2.4), the effective lagrangian can be written

$$\mathcal{L}_{\text{eff}}^{(4)} = \mathcal{L}_{\text{obs}} + \mathcal{L}_{\text{hid}} + \mathcal{L}_{\text{SUGRA}} + O(\epsilon/m_*)\,, \tag{2.6}$$

where $\mathcal{L}_{\text{obs}}$ ($\mathcal{L}_{\text{hid}}$) contains the observable (hidden) fields and covariant couplings to SUGRA, and $\mathcal{L}_{\text{SUGRA}}$ contains the SUGRA kinetic terms and self-couplings. Higher-dimension couplings connecting the hidden and observable sectors are suppressed by a small parameter ϵ compared to the naïvely-expected suppression $1/m_*$. The form of this lagrangian is guaranteed by the higher-dimensional scenario discussed above, but may occur more generally. Equation (2.6) captures what ref. [4] refer to as a "sequestered" sector.

2.2 Supergravity and the conformal compensator

From now on, our discussion will be in the context of the 4-dimensional effective theory given by eq. (2.6). We now review some aspects of 4-dimensional $\mathcal{N} = 1$ SUGRA that are necessary to understand anomaly-mediation.

In the minimal set of auxiliary fields for $\mathcal{N} = 1$ SUGRA, the SUGRA multiplet contains the fields [10]

$$e_m{}^\mu,\ \psi_{\mu\alpha},\ R_\mu,\ H, \tag{2.7}$$

where $e_m{}^\mu$ is the tetrad, $\psi_{\mu\alpha}$ is the gravitino, R_μ is a real auxiliary field and H is a complex auxiliary field. We will be interested in SUGRA backgrounds corresponding to flat space and broken SUSY. In this case, the only nonzero VEV's of the SUGRA multiplet are

$$\langle e_m{}^\mu \rangle = \delta_m{}^\mu\,, \qquad \langle H \rangle = m_{3/2}\,. \tag{2.8}$$

[6]Note that the loop graph is finite and calculable because of the "point splitting" due to the spatial separation of the hidden and observable sectors, and therefore it is an issue whether it has the right sign.

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The fact that $\langle H \rangle$ is equal to the gravitino mass comes from the fermionic terms, which we do not discuss here. Note also that the usual Weyl rescaling of the gravitational fields is not necessary since we work in a fixed background. In this background, we can keep track of the auxiliary field H by introducing a chiral field

$$\Sigma = 1 + \theta^2 H \tag{2.9}$$

with conformal weight $+1$ and demanding conformal invariance of the action [11, 12].[7] It is a non-trivial result that eq. (2.9) preserves a local super-Poincaré symmetry. The field Σ is called the conformal compensator, and acts as a spurion field for the breaking of conformal invariance. For example, if we assign all matter and gauge fields to have vanishing conformal weight, the action for matter and gauge fields in a SUGRA background can be written

$$\begin{aligned}\mathcal{L}_{\text{matt}} = & \int d^2\theta d^2\bar{\theta}\, \Sigma^\dagger \Sigma\, K(\Phi^\dagger e^V, \Phi) + \left[\int d^2\theta\, \frac{1}{2g^2} \operatorname{tr} W^\alpha W_\alpha + \text{h.c.}\right] + \\ & + \left[\int d^2\theta\, \Sigma^3\, W(\Phi) + \text{h.c.}\right] + \text{derivative terms}\,.\end{aligned} \tag{2.10}$$

Note that Σ does not appear in the gauge kinetic term because the conformal weight of $W^\alpha W_\alpha$ is 3. We can define fields with arbitrary conformal weight by rescaling by powers of Σ; for example, the chiral fields $\Phi' = \Sigma\Phi$ have conformal weight $+1$. To obtain the SUGRA scalar potential from this expression, one must integrate out the auxiliary field H, with a constant term in the superpotential adjusted to cancel the vacuum energy.

2.3 Anomaly mediation

We now consider an observable sector coupled to a "sequestered" sector by SUGRA, as discussed above. The observable sector may have higher-dimension M_*-suppressed operators coupling observable fields, but these do not mediate SUSY breaking and can therefore be ignored. The observable sector is therefore well approximated by a renormalizable theory, and we can write

$$\begin{aligned}\mathcal{L}_{\text{obs}} = & \int d^4\theta\, \Sigma^\dagger \Sigma\, Z_a Q_a^\dagger e^V Q^a + \int d^2\theta\, \frac{1}{2g^2} \operatorname{tr} W^\alpha W_\alpha + \text{h.c.} + \\ & + \int d^2\theta\, \Sigma^3 \left(\kappa_a Q^a + \frac{1}{2} m_{ab}\, Q^a Q^a + \frac{1}{3!} \lambda_{abc}\, Q^a Q^b Q^c\right) + \text{h.c.}\end{aligned} \tag{2.11}$$

A constant term in the superpotential is required to cancel the cosmological constant, but this can be regarded as part of $\mathcal{L}_{\text{hid}}$.

Consider the case where the observable sector contains no dimensionful interactions, so $\kappa_a = 0$, $m_{ab} = 0$. In terms of the rescaled fields

$$Q'^a = \Sigma Q^a \tag{2.12}$$

[7] For another approach to SUGRA, see ref. [13].

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the lagrangian can be written

$$\mathcal{L}_{\rm obs} = \int d^4\theta\, Z_a Q'^{\dagger}_a e^V Q'^a + \int d^2\theta\, \frac{1}{2g^2}\,\mathrm{tr}\, W^\alpha W_\alpha + \text{h.c.} + \\ + \int d^2\theta \left(\frac{1}{3!}\lambda_{abc} Q'^a Q'^b Q'^c \right) + \text{h.c.} \tag{2.13}$$

All dependence on the conformal compensator Σ has completely disappeared, so there is no SUSY breaking in the observable sector at the classical level. It is clear that the absence of SUSY breaking is closely connected with the conformal symmetry of the classical action that allows us to scale away the Σ dependence. It should therefore not be surprising that the quantum conformal anomaly gives rise to Σ dependence.

This can be made precise in a number of different ways [4, 7]. We will give a non-perturbative argument based on the 1PI definition of soft mass terms [8, 7]. The origin of the Σ dependence in the quantum theory is the fact that the regulator necessarily introduces a mass scale that breaks conformal symmetry explicitly, and therefore introduces Σ dependence. (Note that we are considering quantum effects in a fixed SUGRA background, so we need not regulate SUGRA itself.) Σ is a spurion for conformal symmetry (with conformal weight $+1$), and so the dependence on Σ is determined by dimensional analysis. This allows us to directly read off the Σ dependence in the 1PI effective action. For example, the 1PI 2-point function for a chiral field Q can be written

$$\Gamma_{\rm 1PI} = \int d^4x \int d^4\theta\, Q^\dagger_a\, \zeta_a(\Box(\Sigma^\dagger\Sigma)^{-1}) Q^a + \cdots, \tag{2.14}$$

where the dependence of ζ on $\Box = \partial^\mu\partial_\mu$ is a manifestation of the conformal anomaly, and the Σ dependence is determined by the fact that Σ is a spurion with conformal weight $+1$. The F terms of Σ give rise to a soft mass term for the scalar component of Q^a, and we obtain

$$m_a^2(\mu) = [\ln\zeta_a(\,-\,\mu^2(\Sigma^\dagger\Sigma)^{-1})]_{\theta^2\bar\theta^2} = -\frac{1}{4}\frac{d\gamma_a}{d\ln\mu} m_{3/2}^2\,, \tag{2.15}$$

where $\gamma_a = d\ln Z_a / d\ln\mu$ and

$$Z_a(\mu) = \zeta_a(\mu^2)\,. \tag{2.16}$$

Equations (2.15) and (2.16) are the definitions of the 1PI running soft mass parameter and wavefunction factor in a superfield coupling scheme [8], and eq. (2.15) follows simply by differentiation. This argument is non perturbative, and shows that eq. (2.15) is an RG-invariant relation that holds at all scales. This means that the anomalous dimensions that determine the soft scalar masses are themselves completely determined by the effective theory at the weak scale, with no dependence on the underlying fundamental theory!

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Similar arguments can be given for gaugino masses [4, 7] and A terms [7] and we obtain

$$m_\lambda = \frac{\beta(g^2)}{2g^2} m_{3/2}\,, \qquad B_a = \frac{1}{2}\,\gamma_a\, m_{3/2}\,, \tag{2.17}$$

where $\beta(g^2) = dg^2/d\ln\mu$ and B_a determines the A terms via

$$\Delta\mathcal{L} = \frac{1}{3!}\lambda_{abc}(B_a + B_b + B_c)\tilde{Q}^a\tilde{Q}^b\tilde{Q}^c + \text{h.c.}\,, \tag{2.18}$$

where $\tilde{Q}$ are the scalar components of Q.

3. Slepton masses

We now consider the problem of negative slepton masses. The signs of the soft masses are determined by the signs of anomalous dimensions. Schematically, the anomalous dimension for a chiral field has 1-loop contributions

$$\gamma \sim \frac{1}{16\,\pi^2}\left(-\lambda^2 + g^2\right). \tag{3.1}$$

The scalar masses therefore have signs

$$m^2 \sim -m_{3/2}^2\left(\frac{\partial\gamma}{\partial\lambda}\beta_\lambda + \frac{\partial\gamma}{\partial g}\beta_g\right) \sim +\left(\frac{m_{3/2}}{16\,\pi^2}\right)^2\Big[+\lambda(\lambda^3 - \lambda g^2) - g(\pm g^3)\Big], \tag{3.2}$$

where $\beta_g \sim \pm g^3/(16\pi^2)$. We see that if we neglect the effects of the Yukawa couplings, an asymptotically free gauge group gives a positive scalar mass-squared, while gauge groups that are not asymptotically free give a negative mass-squared. Since the lepton Yukawa couplings are small in the MSSM, and the leptons are charged only under the non-asymptotically free groups $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, the slepton masses are negative. The squark masses obtain large positive contributions from $\mathrm{SU}(3)_C$, and are not problematic.

In the following subsections, we will explore extensions of the MSSM that can give positive slepton masses. In the next section, we will address electroweak symmetry breaking.

3.1 New lepton Yukawa couplings

Perhaps the simplest way to obtain positive slepton masses is to extend the MSSM to include new Yukawa couplings involving leptons. This requires new fields beyond those present in the MSSM. (The new Yukawa couplings must be order 1 for *all* lepton fields in order to overcome the negative gauge contribution to the slepton masses. We therefore cannot make use of the R-parity violating terms allowed in the MSSM.) A simple possibility is to introduce an additional pair of Higgs doublets H_u' and H_d' with superpotential couplings

$$\Delta W = (y_e')_{jk}\, L_j H_d'\, \bar{E}_k\,. \tag{3.3}$$

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Leptoquarks are another simple possibility, but we will focus on extra Higgs doublets. The fields $H'_{u,d}$ must have masses at or below the electroweak scale in order to contribute to the anomalous dimensions of the lepton fields at the weak scale and change the sign of the slepton masses.

There are a number of issues that arise immediately when we consider Yukawa couplings of this form: lepton flavor violation, mixing between H'_d and H_d, and gauge coupling unification. We now address each of these in turn.

We first consider FCNC's. Without special assumptions about the flavor structure, the Yukawa couplings will not be diagonal in the basis that diagonalizes the lepton mass matrix. This will give a tree-level contribution to $\mu^\pm \to \mathrm{e}^\pm \mathrm{e}^+ \mathrm{e}^-$ from exchange of the scalar components of H'_d. At scales below the mass of the new field, this can be parameterized by the effective interaction

$$\mathcal{L}_{\text{int}} \sim \frac{{y'_e}^2}{m^2_{H'd}} (\bar{\mu} e)(\bar{e} e) + \text{h.c.} \tag{3.4}$$

The experimental limit $\Gamma(\mu^\pm \to \mathrm{e}^\pm \mathrm{e}^+ \mathrm{e}^-)/\Gamma_{\text{tot}} < 1.0 \times 10^{-12}$ [14] gives a bound

$$m_{H'd} \gtrsim y'_e \cdot (100\,\text{TeV})\,. \tag{3.5}$$

(The process $\mu^\pm \to \mathrm{e}^\pm \gamma$ gives somewhat weaker bounds.) Since y'_e must be of order 1, this forces the soft mass for H'_d to unnaturally large values that can destabilize the potential for the other Higgs fields.

We are therefore forced to assume that there is additional non-trivial flavor structure in the lepton sector. Perhaps the simplest possibility is to assume that there is a flavor symmetry that forces the lepton Yukawa couplings to be diagonal.[8] It may appear that such symmetry forbids neutrino mixing, but this need not be the case. Currently, the most convincing evidence for neutrino mixing comes from the solar and atmospheric neutrino anomalies, which are purely disappearance effects. These can be explained by mixing with sterile neutrinos that carry lepton family numbers.

Even mixing among "active" neutrino flavors does not preclude the existence of approximate lepton family number conservation for charged leptons, which can emerge as accidental symmetries. For example, assume that the $(\mathbb{Z}_2)^3$ flavor symmetry is broken by a spurion S with total lepton number $+2$ in a model with right-handed neutrinos. (For example, S may be proportional to VEV's of fields with lepton number $+2$.) This gives a Majorana mass for the right-handed neutrinos:

$$W = y_{ej} L_j H_d \bar{E}_j + y_{\nu j} L_j H_u \bar{N}_j + \frac{1}{2} M S_{jk} \bar{N}_j \bar{N}_k\,. \tag{3.6}$$

We assume that $M \gg M_W$. Below the scale M, this gives an effective interaction

$$W_{\text{eff}} = y_{ej} L_j H_d \bar{E}_j - \frac{y_{\nu j} y_{\nu k}}{2M} (S^{-1})_{jk} (L_j H_u)(L_k H_u) + \cdots \tag{3.7}$$

[8]The symmetry may also be approximate, but we take it to be exact for simplicity.

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The $1/M$ term gives small Majorana neutrino masses of order $y_\nu^2 v^2/M$; this is the standard see-saw mechanism. The low-energy theory contains no modification of the diagonal form of the charged lepton Yukawa matrices because these conserve lepton number.

We now turn to the issue of H'_d–H_d mixing. This turns out to be the most severe constraint on this scenario. Because H'_d has identical quantum numbers as H_d and both fields couple to leptons, these fields will mix at one loop. Because of this, the anomaly-mediated soft masses contain off-diagonal terms

$$\Delta V_{\rm soft} = \Delta m^2 (H_d^\dagger H'_d + \text{h.c.})\,, \tag{3.8}$$

with $\Delta m^2 \sim M_{\rm SUSY}^2 y_\tau {y'_e}^3$. These terms are dangerous because they give a tadpole for H'_d, resulting in $\langle H'_d \rangle / \langle H_d \rangle \sim y_\tau / y'_e$. This in turn gives a contribution to the electron mass

$$\Delta m_e \sim y'_e \langle H'_d \rangle \sim m_\tau\,. \tag{3.9}$$

The observed value of the electron mass can be recovered only at the expense of an unnatural fine-tuning of order $m_e/m_\tau \sim 3 \times 10^{-4}$. This is clearly unacceptable.

We can solve this problem in a manner consistent with 't Hooft naturalness by adding 3 new Higgs fields H'_{dj}, H'_{uj} ($j = 1, 2, 3$). The $(\mathbb{Z}_2)^3$ symmetry allows us to write the superpotential couplings

$$W = y_{ej}\, L_j H_d\, \bar{E}_j + y'_{ejk}\, L_j H'_{dk}\, \bar{E}_j\,. \tag{3.10}$$

In order to suppress Higgs mixing we assume the existence of an *approximate* $(\mathbb{Z}_2)'^3$ symmetry, where under $(\mathbb{Z}_2)'_1$ (for example) L_1, H'_{d1}, and H'_{u1} are odd and all other fields are even.[9] If the symmetry were exact, it would force the couplings y'_{ejk} to be exactly diagonal. This symmetry is violated by the electron Yukawa coupling (since H_d and $\bar{E}_1$ are even under $(\mathbb{Z}_2)'_1$). There are 2-loop effects proportional to $y_e^2 {y'^2}_e$ that contribute to mixing between e.g. H'_{d1} and H'_{d2}, and so we naturally have

$$y'_{ejk} = y'_{ej}\, \delta_{jk} + O(y_{ej} y_{ek})\,, \quad y'_{ej} \sim 1. \tag{3.11}$$

With these assumptions, the mixing between e.g. H'_{d1} and H_d is controlled by y_{e1} at one loop, and so does not give an unnaturally large contribution to the electron mass.

It may appear that the assumption of approximate lepton flavor symmetries is somewhat artificial. However, the small observed values of the lepton Yukawa couplings are only natural if approximate flavor symmetries are present. They may arise as accidental symmetries, or due to a hierarchical breaking of flavor symmetries in a more fundamental theory.

[9]Note that this allows a "μ term" $H'_{u1} H'_{d1}$.

A significant drawback of this scenario is that the only natural way to preserve one-step gauge coupling unification is to add 3 color triplets, so that we are adding 3 $\mathbf{5} \oplus \bar{\mathbf{5}}$'s of SU(5). But then the 1-loop $SU(3)_C$ beta function vanishes, and squark masses are too small.[10] Of course, this does not mean that the model is incompatible with the general idea of unification, since one can have intermediate thresholds and/or non-minimal GUT groups. However, we do give up a simple explanation of the striking fact that the simplest possible one-step unification appears to work in the MSSM.

3.2 Compositeness at the weak scale

We have seen that adding new lepton Yukawa couplings requires special flavor structure in the lepton sector. Since gauge interactions naturally conserve flavor, it is natural to consider the possibility that gauge interactions give rise to positive contributions to slepton masses. The difficulty is that this requires the gauge group to be asymptotically free, and hence non abelian.

One possibility is that the leptons are composite with a compositeness scale Λ_{comp} near M_{SUSY}. That is, we assume that there is a new asymptotically-free gauge group that gets strong at the scale Λ_{comp}, whose non-perturbative dynamics produces light fermions with the quantum numbers of the observed leptons. The scale Λ_{comp} must be low enough so that the non abelian gauge group dominates the anomalous dimensions of the lepton degrees of freedom at the scale where their masses are generated. This scenario therefore requires lepton compositeness near the weak scale.

This scenario is severely constrained by searches for deviations in cross sections at colliders ("compositeness searches" [14]). In the effective theory below Λ_{comp}, we expect 4-fermion operators of the form

$$\mathcal{L}_{\text{int}} \sim \frac{(4\pi)^2}{\Lambda_{\text{comp}}^2} (\bar{e}e)^2 \,. \tag{3.12}$$

The factor of $(4\pi)^2$ is inserted based on "naïvedimensional analysis" [16]: Λ_{comp} is defined as the scale at which the underlying theory becomes strongly interacting in the sense that loop corrections are unsuppressed, and this should coincide with the scale at which the effective theory becomes strong. With 4-fermion couplings of this strength, current experimental limits give [14]

$$\Lambda_{\text{comp}} \gtrsim 18\,\text{TeV}\,. \tag{3.13}$$

This bound does not necessarily translate into a large value for M_{SUSY}, because the gauge coupling g_{comp} can be large at the scale where the slepton masses are generated.

[10] In fact, when we consider electroweak symmetry breaking, we find that even if we add two $\mathbf{5} \oplus \bar{\mathbf{5}}$'s, obtaining sufficiently large squark masses requires additional fine tuning.

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To estimate this, we use the 1-loop RG equations for $g_{\rm comp}$,

$$\mu \frac{d}{d\mu} \left(\frac{1}{g_{\rm comp}^2} \right) = \frac{b}{8\pi^2}, \tag{3.14}$$

and define $\Lambda_{\rm comp}$ to be the scale where $g_{\rm comp}^2 \sim 16\pi^2/b$. Demanding that the slepton mass $m_{\tilde{\ell}} \sim g_{\rm comp}^2 M_{\rm SUSY}$ is larger than $\Lambda_{\rm comp}$ then gives

$$M_{\rm SUSY} \gtrsim \frac{\Lambda_{\rm comp}}{g^2(m_{\tilde{\ell}})} \sim \frac{b}{8\pi^2} m_{\tilde{\ell}} \left(\frac{1}{2} + \ln \frac{m_{\tilde{\ell}}}{\Lambda_{\rm comp}} \right). \tag{3.15}$$

For example, for $\Lambda_{\rm comp} \sim m_{\tilde{\ell}} \sim 20\,{\rm TeV}$, $b \sim 5$ we obtain $M_{\rm SUSY} \sim 600\,{\rm GeV}$. There are clearly large theoretical uncertainties in these estimates, but they show that this scenario is not impossible. (Note that such heavy sleptons do not give large naturalness-spoiling contributions to the Higgs mass because the lepton Yukawa couplings are small.)

One theoretical puzzle that arises in this scenario is the question of why the compositeness scale should be so close to $M_{\rm SUSY}$, since these scales have a different origin. (In the scenario of ref. [4], the SUSY breaking sector that sets the scale $M_{\rm SUSY}$ is literally in another world!) However the near coincidence of these scales may be natural because the scalars and gauginos charged under the compositeness gauge group get masses of order $m_{\tilde{\ell}} \sim g_{\rm comp}^2 M_{\rm SUSY}$. The beta function of the compositeness gauge group is more negative below this scale. Then the running coupling $g_{\rm comp}$ may therefore become strong rapidly, naturally explaining why $\Lambda_{\rm comp}$ is close to $M_{\rm SUSY}$. This mechanism will be especially efficient if the compositeness gauge coupling is “walking” above the compositeness scale. A smaller beta function for the compositeness gauge group also suppresses the soft masses of the scalars and gauginos charged under the compositeness group, but this may be compensated by larger group theory factors. For example, we can envision a situation where the beta functions of the composite gauge group is of order the MSSM beta functions, but the group theory factors make some scalars charged under the compositeness heavier than MSSM scalars. A definite answer to the question of the naturalness of such a scenario is difficult to obtain without a more detailed understanding of the compositeness dynamics, and will not be attempted here.

Having composite leptons and elementary quarks may be difficult to reconcile with one-step gauge coupling unification. The simplest way to ensure gauge coupling unification is to construct a model in which quarks as well as leptons are composite, and that the “preons” as well as the composites occur in complete SU(5) multiplets.

Finding a model that embodies this mechanism is clearly non trivial. In particular, the compositeness dynamics is inherently non supersymmetric below the scale $m_{\tilde{\ell}}$, and therefore not under theoretical control. However, we regard this scenario as an interesting (albeit speculative) possibility.

3.3 Horizontal gauge symmetries

Another possible way to couple leptons to a non abelian gauge symmetry is to assume that lepton flavor symmetry is gauged. Since the gauge group must be unbroken near the weak scale (in order for its anomalous dimensions to affect the slepton masses), this forces us to build a model of flavor at the weak scale. This theory contains gauge bosons that change flavor quantum numbers, and therefore does not naturally suppress FCNC's. For example, the process $\mu^{\pm} \to \mathrm{e}^{\pm}\mathrm{e}^{+}\mathrm{e}^{-}$ gives a bound on the mass of the horizontal gauge boson $M_{\mathrm{hor}} \gtrsim 85\,\mathrm{TeV}$ (assuming a horizontal gauge coupling of order g_2 and no alignment). This requires M_{SUSY} to be of comparable size, and necessitates fine tuning in order to obtain electroweak symmetry breaking.[11] We will not consider this possibility further.

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4. Electroweak symmetry breaking

We now turn to electroweak symmetry breaking. One crucial issue is the μ term. A tree-level μ term in the observable sector can be written

$$\Delta\mathcal{L} = \int d^2\theta\, \Sigma^3 \mu H_u H_d + \mathrm{h.c.} \tag{4.1}$$

where Σ is the conformal compensator discussed in subsection 2.2. After scaling out the compensator, we see that $B\mu = \mu m_{3/2} \sim 16\pi^2 M_{\mathrm{SUSY}}\mu$, which is too large since other soft masses are of order M_{SUSY}. This is a direct result of having a dimensionful parameter in the observable sector, which breaks conformal invariance at tree level and ruins the anomaly-mediation result that all soft masses are loop suppressed. An attempt to use the Giudice-Masiero mechanism [9] requires extra interactions between the hidden and observable sector, and does not give $\mu^2 \sim B\mu \sim M_{\mathrm{SUSY}}^2$ unless parameters are adjusted to special values. Since the original "μ problem" is precisely to explain why an arbitrary parameter (a tree-level μ term in the MSSM) has a special value ($\sim M_W$) we do not regard this as an attractive possibility.

Clearly the most natural possibility is that an effective μ term is generated by SUSY breaking at the weak scale. We are led to the idea that the μ term arises as the VEV of a singlet field S with a superpotential coupling SH_uH_d. If the scalar mass-squared term for S is negative, this will naturally give $\langle S\rangle \sim M_{\mathrm{SUSY}}$. The idea that the μ term arises from the VEV of a singlet at the weak scale has been considered by many authors [15]. We will also assume that the slepton mass problem is solved by the addition of 3 extra pairs of Higgs doublets H'_{uj}, H'_{dj} ($j = 1, 2, 3$) and triplets $T, \bar{T}$, as discussed in subsection 3.1. We also add 3 additional singlets R_j ($j = 1, 2, 3$),

[11] This would be somewhat ameliorated if the gauge coupling of the horizontal gauge group were larger. However, in the limit where the gauge coupling approaches the perturbative limit, the model becomes a composite model that violates flavor.

which we take to be odd under the corresponding factor of the approximate $(\mathbb{Z}_2)'^3$ For simplicity, we also assume that the $(\mathbb{Z}_2)^3$ symmetry discussed in subsection 3.1 above is exact. The most general renormalizable superpotential invariant under the (exact and approximate) flavor symmetries is then

$$\begin{aligned} W = {} & \lambda_H S H_d H_u + \frac{\kappa}{3} S^3 + \lambda_T S T \bar{T} + \frac{b_j}{2} S R_j^2 + \\ & + \lambda'_{Hj} S H'_{dj} H'_{uj} + a_{uj} R_j H'_{dj} H_u + a_{dj} R_j H_d H'_{uj} + (y'_e)_j H'_{dj} L_j \bar{E}_j + \\ & + (y_u)_{jk} Q_j H_u \bar{U}_k + (y_d)_{jk} H_d Q_j \bar{D}_k \,. \end{aligned} \tag{4.2}$$

The conventional lepton Yukawa couplings

$$\Delta W = y_{ej} H_d L_j \bar{E}_j \tag{4.3}$$

are invariant under $(\mathbb{Z}_2)^3$, but break the approximate $(\mathbb{Z}_2)'^3$ symmetry, so we expect additional terms in the superpotential suppressed by factors proportional to lepton Yukawa couplings. In particular, there are 1- and 2-loop diagrams that give rise to mixings: H'_{dj}–H_d, H'_{uj}–H_u, and R_j–S. These will give rise to mixing masses, e.g.

$$\Delta V_{\text{soft}} = \Delta m^2_{SRj} (S^\dagger R_j + \text{h.c.}) \tag{4.4}$$

with

$$\Delta m^2_{SRj} \sim M^2_{\text{SUSY}}\, a_{uj}\, y'_{ej}\, y_{ej}\, \lambda_H \,. \tag{4.5}$$

This will give rise to a tadpole for R_j; if $m^2_R \sim M^2_{\text{SUSY}}$ is positive, we have $\langle R_j \rangle \sim a_{uj}\, y'_{ej}\, y_{ej}\, \lambda_H \langle S \rangle$. These effects are all suppressed by small lepton Yukawa couplings, and can therefore be neglected for purposes of discussing electroweak symmetry breaking.

The soft SUSY breaking terms are completely determined by the dimensionless couplings in the superpotential, together with the gauge couplings. (Explicit formulas are given in the appendix.) We look for a solution with

$$\langle S \rangle \sim \langle H_u \rangle \sim \langle H_d \rangle \sim M_W \,, \tag{4.6}$$

with all other VEV's small. (As discussed above, if the soft masses of $H'_{u,d}$ and R are positive and of order M_{SUSY}, they will get small VEV's suppressed by lepton Yukawa couplings.) The role of the various terms in the superpotential (4.2) are as follows:

The triplets $T, \bar{T}$ are required because the term $ST\bar{T}$ gives an important negative contribution to the mass-squared of S from the term (see (3.2))

$$m^2_S = M^2_{\text{SUSY}} \lambda^2_T (15\lambda^2_T - 16 g^2_3) + \cdots . \tag{4.7}$$

This is required in order for $\langle S \rangle$ to be sufficiently large. (The term $S H_u H_d$ gives a tadpole for S, but results in small values for $\langle S \rangle$, and therefore a small μ term.)

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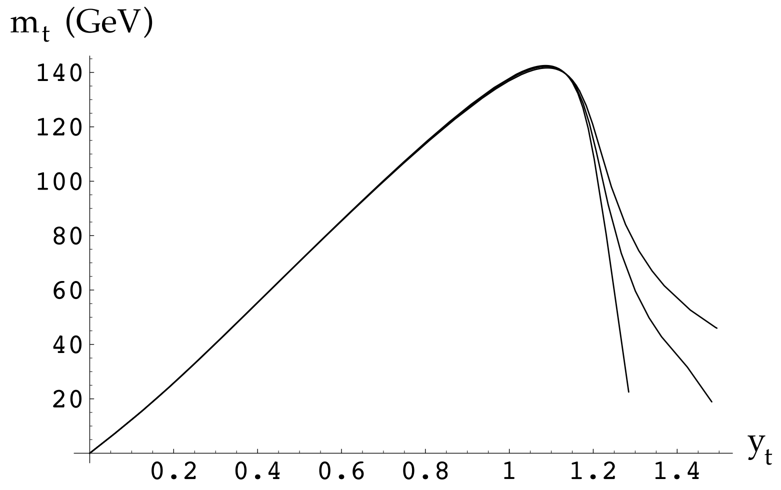

Figure 2: Top quark mass as a function of y_t for different values of λ_T. From the upper curve to the lower, the values of λ_T are 0.35, 0.25, and 0.15. The curves end at a finite value of y_t because the model no longer breaks electroweak symmetry. The other parameters are $\lambda_H = 0.15$, $\lambda_T = 0.15$, $\lambda'_H = 0.15$, $\kappa = 0.3$, $a_d = 0$, $a_u = 0$, $b = 0$, $y'_e = 0.95$, $y_t = 1$, $y_b = 0.1$, $y_\tau = 0.05$.

The singlets R_j are required to obtain electroweak symmetry breaking together with the observed value of the top quark mass, as we now explain. The SU(2) and U(1) gauge couplings give negative contributions to both m^2_{Hu} and m^2_{Hd} that can induce electroweak symmetry breaking. In addition, m^2_{Hu} receives a contribution from the top Yukawa coupling

$$m^2_{Hu} = 18 y_t^2 (y_t^2 - \frac{8}{9} g_3^2) M^2_{\mathrm{SUSY}} + \cdots . \tag{4.8}$$

In the small $\tan\beta$ regime where $y_b, y_\tau \ll 1$, y_t is the only parameter that differentiates between m^2_{Hu} and m^2_{Hd}. Therefore, $\tan\beta$ is determined almost completely by y_t, and so the top mass is determined almost completely by y_t. This is illustrated in figure 2. By varying parameters, we find that it is not possible to get a top mass above $\sim 145\,\mathrm{GeV}$ in the small $\tan\beta$ regime. This is illustrated in figure 2. For $y_t \lesssim g_3$, the contribution in eq. (4.8) is negative and therefore $\tan\beta > 1$. For $y_t \gtrsim g_3$ we obtain $\tan\beta < 1$, and as y_t increases both $\tan\beta$ and m_t decrease until the electroweak symmetry breaking is lost. The introduction of the singlets R_j with couplings $RH_uH'_d$ and $RH_dH'_u$ give additional contributions to m^2_{Hu} and m^2_{Hd}, thus eliminating the correlation between y_t and $\tan\beta$ just described.

The above discussion assumes that $\tan\beta \sim 1$. We also explored the possibility of large $\tan\beta$ in the model without the singlets R_j, but find solutions only with large fine-tuning in several parameters. The reason for this is that $m^2_{Hu}/M^2_{\mathrm{SUSY}}$ is bounded from below by the anomalous dimension formulas. This means that the only way to obtain large $\tan\beta$ is to have small $B\mu/M^2_{\mathrm{SUSY}}$, and hence small μ/M_{SUSY}. But since

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there is a chargino mass of order μ, this means that $M_{\rm SUSY} \gg v$, which can only be obtained by extreme fine tuning. We find solutions only with fine tuning $\sim 10^{-3}$ in both y_t and y_b.

The couplings $RH_uH'_d$ and $RH_dH'_u$ have another purpose, besides allowing us to reproduce the observed top mass. Even if we could somehow live with $m_t \simeq 145\,\text{GeV}$, we would not have a viable solution without the singlet R. The problem is that for "generic" choices of the parameters, the Higgs VEV is $v \gtrsim 5M_{\rm SUSY}$, where $v = 174\,\text{GeV}$ is required to reproduce the correct values of M_W and M_Z. This results in a small value of $M_{\rm SUSY}$, and hence many superpartner masses are too small. This problem can be cured only by adjusting parameters so that the potential is near the critical point for electroweak symmetry breaking (so that $v \sim M_{\rm SUSY}$). However, this is not possible without the couplings a_u and a_d.

This explains why we need the couplings $RH_uH'_d$ and $RH_dH'_u$: they allow us to control m^2_{Hu} and m^2_{Hd} independently, eliminating the correlation between y_t and $\tan\beta$ discussed above. We must introduce new singlets R_j for this purpose rather than using a term $SH_uH'_d$ because we must avoid a tadpole $\langle S\rangle\langle H_u\rangle H'_d$ that would otherwise give a large VEV for H'_d.

We now discuss some important features of the superpotential eq. (4.2) that allow this model to be realistic.

- The terms proportional to λ_H, λ'_H, λ_T, and b give rise to effective "μ terms" that give masses proportional to $\langle S\rangle$ for the fermion fields of $H_{u,d}$, $H'_{u,d}$, T, $\bar{T}$, and R. We therefore need all of these couplings to be non zero.

- As already discussed above, y_t and λ_T give important negative contributions to m^2_{Hu} and m^2_S, respectively (see eq. (3.2)), which allow $\langle H_{u,d}\rangle, \langle S\rangle$ to have realistic values.

- The couplings $SH'_uH'_d$, $RH_uH'_d$, and $RH_dH'_u$ give a positive contribution to $m^2_{H'u}$ and $m^2_{H'd}$, necessary to obtain $\langle H'_u\rangle \simeq \langle H'_d\rangle \simeq 0$.

- The coupling κ is required to break a U(1) Peccei-Quinn symmetry that would otherwise give rise to a weak-scale axion.

- R-parity can be extended to the new fields above by taking the scalar components of S, R, $H'_{u,d}$, T, and $\bar{T}$ to have R-parity $+1$, while their fermion components have R-parity -1. This means that this model is of the conventional R-parity conserving type, with a stable LSP.

- As discussed so far, the fields $T, \bar{T}$ carry an exactly conserved quantum number, so the lightest particle carrying this quantum number is stable. The constraints on such particles from direct experiment [17] and cosmology [18] are quite

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stringent. However, these are easily avoided by small Yukawa couplings that allow the T to decay:

$$\Delta W = \epsilon_j R_j T \bar{D}_j \,. \tag{4.9}$$

These violate flavor symmetries, but e.g. $\epsilon_1 \sim 10^{-5}$ is suffucient for a T particle with mass $\sim 150\,\mathrm{GeV}$ to have decay with $c\tau \sim 1$ mm. This is safe from the constraints, while still being consistent with the existence of the approximate flavor symmetries discussed above.

As with all realistic SUSY models, some degree of fine tuning is required to avoid light superpartner masses. In the present model, this arises because for "generic" values of the parameters, we have $v \gtrsim 5M_{\mathrm{SUSY}}$, resulting in a small value for M_{SUSY} and hence light superpartner masses. We require $M_{\mathrm{SUSY}} \gtrsim 100\,\mathrm{GeV}$, which means that we need $v/M_{\mathrm{SUSY}} \lesssim 1$.

The reason for the "generically large" Higgs VEV is that the quartic terms in the Higgs potential are small. The coefficient of the quartic term arising from the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ D terms is $1/8(g_1^2 + g_2^2) \simeq 0.066$; there is also a quartic term proportional to λ_H^2 from the F terms, but λ_H cannot be ~ 1 because this would give a too-large positive contribution to m_S^2, resulting in $\langle S \rangle \simeq 0$. We must therefore tune the parameters so that the Higgs potential is near the critical point where $\langle H_u \rangle = \langle H_d \rangle = 0$, so that v is small in units of M_{SUSY}. Near the critical point, the Higgs VEV is determined by a formula of the form $v^2 \sim m^2/\lambda$ where m^2 is the coefficient of a quadratic term and λ is the coefficient of a quartic term.[12] Since m^2 is analytic in the couplings, near the critical point

$$v \sim (c - c_{\mathrm{crit}})^{1/2} \,, \tag{4.10}$$

where c is a coupling that acts as the control parameter. This is illustrated in figure 3.

Because the tree-level quartic terms in the Higgs potential are small, loop corrections to these terms can be important. In addition to the usual y_t^4 corrections of the MSSM, there are large $a_{u,d}^4$ corrections arising from the Yukawa terms $a_u R H_u H_d'$ and $a_d R H_d H_u'$. These generate non-supersymmetric corrections to the quartic terms in the Higgs potential

$$\Delta V = \Delta\lambda_u (H_u^\dagger H_u)^2 + \Delta\lambda_d (H_d^\dagger H_d)^2 \,, \tag{4.11}$$

with

$$\begin{aligned} \Delta\lambda_u &\simeq \frac{1}{8\pi^2}\left[3\, y_t^4 \ln\frac{m_{\tilde{t}}}{m_t} + 3a_u^4 \ln\frac{m_R}{m_{\tilde{R}}}\right], \\ \Delta\lambda_d &\simeq \frac{1}{8\pi^2} 3\, a_d^4 \ln\frac{m_R}{m_{\tilde{R}}} \,, \end{aligned} \tag{4.12}$$

[12] The potential also has cubic terms, but they are less important near the critical point.

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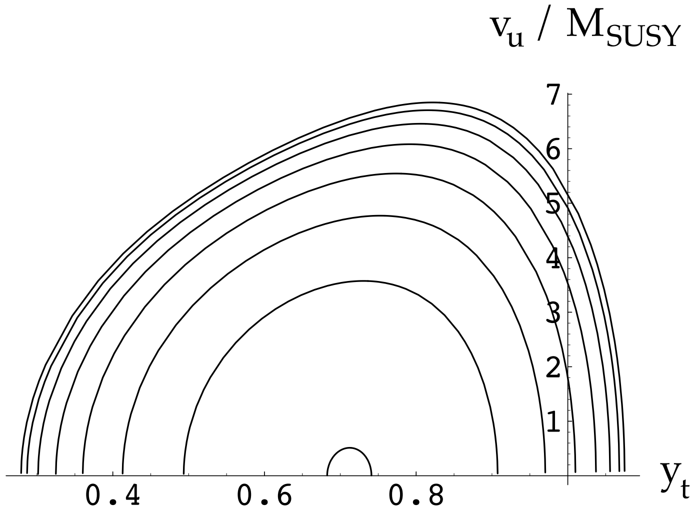

Figure 3: Fine tuning of electroweak symmetry breaking. The plot shows v_u as a function of y_t for values of a_{u3} ranging from 0.1 (upper curve) to 0.8 (lower curve). The values of the other coupling constants are as in the example in section 4.

where R ($\tilde{R}$) is the scalar (fermion) component of the superfield R. (We have taken universal values for a_{uj}, a_{dj} for simplicity.) These corrections increase the mass of the neutral Higgs boson, and also reduce the amount of fine-tuning required for a realistic solution.

We have not attempted to explore the full parameter space of this model, but we have found regions of the parameter space with realistic solutions. The solutions we find have $\langle S \rangle \sim 7 M_{\rm SUSY}$ and small λ_H; this gives a sufficiently large μ term while avoiding large positive contributions to m_S^2, m_{Hu}^2, and m_{Hd}^2 from λ_H. This requires small values of κ, and so we are driven to a regime where the scalar component of S is light (it is a pseudo-Nambu-Goldstone boson of the Peccei-Quinn symmetry). This means that the Higgs sector differs significantly from the MSSM Higgs sector, and the lightest CP-even Higgs boson can be heavier than M_Z even without the radiative corrections discussed above. Also, because of the presence of extra neutral fermions and moderate μ terms, the neutralino-chargino degeneracy discussed in refs. [4, 7] is not realized in our model. We have checked that all scalar mass-squared terms are positive at these points.

As an example, we take $\lambda_H = 0.15$, $\lambda_T = 0.17$, $\lambda'_H = 0.15$, $\kappa = 0.075$, $a_d = 1$, $a_u = 0.5$, $b = 0.05$, $y'_e = 0.95$, $y_t = 1$. (We take universal values for $\lambda'_{Hj}, a_{uj}, a_{dj}, b_j$, and set $y_b = y_\tau = 0$.) In this case, we find

$$\frac{v_u}{M_{\rm SUSY}} = 1.0\,, \qquad \frac{v_d}{M_{\rm SUSY}} = 0.34\,, \qquad \frac{\langle S \rangle}{M_{\rm SUSY}} = 6.9\,. \tag{4.13}$$

This gives $M_{\rm SUSY} = 160\,\text{GeV}$, and reproduces the observed top quark mass with $\tan\beta = 3.1$. For this solution, the lightest neutralino is mainly R fermion, and chargino and neutralino are not degenerate: $m_{\tilde{\chi}_1^0} = 87\,\text{GeV}$, $m_{\tilde{\chi}_1^\pm} = 129\,\text{GeV}$. Near

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degeneracy of the lightest neutralino and chargino is a generic feature of an anomaly-mediated spectrum in the MSSM provided there are no additional neutral fermions and the μ term is sufficiently large [4, 7]. In the present case, both of these conditions are violated ($\mu = 160\,\text{GeV}$).

The lightest neutral CP-even Higgs has mass 91 GeV, and contains a significant admixture of S scalar. The lightest neutral CP-odd Higgs has mass 180 GeV and is mainly S scalar. The sleptons have masses ranging from 130 GeV (for sneutrinos) to 310 GeV (for right-handed selectrons), while squarks of the first two generations have masses near 430 GeV. The gluino mass is 380 GeV. There are also colored bosons from the fields $T, \bar{T}$ with mass 170 GeV. Of course, this point is only meant as an example, but it shows that the sparticle spectrum can be quite conventional apart from the large number of additional states compared to the MSSM.

We now discuss the fine tuning. The most sensitive dependence of M_Z on fundamental parameters is[13]

$$\frac{\partial \ln M_Z}{\partial \ln y_t} = 77\,, \qquad \frac{\partial \ln M_Z}{\partial \ln a_u} = 13\,, \tag{4.14}$$

Here, y_t is the running parameters at the weak scale. If we express the fine-tuning in terms of fundamental parameters at a higher scale, the fine-tuning is significantly less due to the infrared quasi-fixed point for y_t [21]. For example, the sensitivity is reduced by a factor of 10 in terms of $y_t(\mu \sim 10^8\,\text{GeV})$. We conclude that the fine tuning in this model is no worse than other SUSY models such as gauge mediation of minimal supergravity.

Since the parameters we have chosen are somewhat fine-tuned in terms of parameters at the weak scale, the VEV's may be sensitive to loop corrections. However, the existence of a critical point where $\langle H_{u,d} \rangle \to 0$ should survive the inclusion of loop effects, and there will be realistic solutions when the parameters are adjusted to appropriate values. We believe that our analysis is therefore sufficient to conclude that the model can be realistic.

5. Conclusions

We have investigated the possibility that supersymmetry breaking is communicated to the observable sector by the recently-discovered mechanism of anomaly mediation [4, 7]. This is automatically the leading mechanism if the lagrangian of the world has the "sequestered" form

$$\mathcal{L} = \mathcal{L}_{\text{obs}} + \mathcal{L}_{\text{hid}} + \mathcal{L}_{\text{SUGRA}} + O(\epsilon/m_*)\,, \tag{5.1}$$

[13]This is the measure of fine-tuning proposed in ref. [19]. It has been emphasized that this measures sensitivity, which does not imply fine-tuning if the sensitivity is high for all *a priori* allowed parameters [20]. In the present case, for generic values of the input parameters the sensitivity is much less (see figure 3), and so the large sensitivity is a sign of fine-tuning.

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where $\mathcal{L}_{\text{obs}}$ ($\mathcal{L}_{\text{hid}}$) contains only the observable (hidden) sector fields and their couplings to supergravity, and $\mathcal{L}_{\text{SUGRA}}$ contains the supergravity kinetic terms and self-interactions. Higher-dimension operators that directly connect the hidden and observable sectors are suppressed by an additional small parameter ϵ compared to the naïvely-expected suppression $1/m_*$, where m_* is the reduced Planck scale. Such a mechanism is naturally realized if the hidden sector is a parallel universe in higher dimensions [4]. In that case $\epsilon \sim e^{-M_* R}$, where M_* is the higher-dimensional Planck scale and R is the size of the "large" extra dimensions.

This mechanism automatically solves the SUSY flavor problem, and is also theoretically very appealing. However, it suffers from several glaring phenomenological problems: slepton masses are negative, and the μ problem is more difficult to solve. Previous discussions have assumed additional direct couplings between the hidden and observable sector, requiring special choices of parameters to ensure that all SUSY breaking is at the weak scale. In this paper, we have constructed a realistic model with *no* direct couplings between the hidden and observable sector. This means that *all* dimensionful parameters in the observable sector arise from anomaly mediation, and are therefore determined by anomalous dimensions at the weak scale. In this type of model, the superpartner masses are directly determined by dimensionless couplings of fields at the weak scale; both the masses and couplings are experimentally accessible, so these relations are in principle testable. These models also generally contain a large number of additional charged particles near the weak scale that are subject to experimental study. Our model extends the minimal supersymmetric standard model by 3 pairs of Higgs doublets, 1 vector-like pair of color triplets, and 4 new singlets. These fields are constrained by approximate discrete flavor symmetries in order to avoid lepton flavor-changing neutral currents and unwanted mixing. The μ term is generated by the vacuum expectation value of a singlet. The model is tightly constrained, and it is non-trivial that the tree-level and loop effects can combine to give a realistic spectrum. This class of models provides a theoretically well-motivated and phenomenologically interesting framework for physics beyond the standard model.

Note Added: while completing this paper, we received an interesting paper by A. Pomerol and R. Rattazzi [22] that considers a different mechanism that may make anomaly-mediation realistic. We also received refs. [23, 24], which consider experimental signatures of the nearly degenerate neutralino/chargino.

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A. Useful formulas

As discussed in the main text, we ignore mixing between $H_{u,d}$ and $H'_{u,d}$ for simplicity, so we look for solutions with $\langle H'_{u,d}\rangle = 0$. The relevant terms in the superpotential for electroweak symmetry breaking are then

$$W = \lambda_H S H_d H_u + \frac{\kappa}{3} S^3 \,. \tag{A.1}$$

We can choose λ_H and κ to be real without loss of generality. The most general solution up to $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ rotations can be parameterized as

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle H_d \rangle = \begin{pmatrix} v_d \cos\theta \\ v_d \sin\theta \end{pmatrix} e^{i\alpha}, \qquad \langle S \rangle = x e^{i\gamma}, \tag{A.2}$$

where v_u, v_d, and x are all real. The F terms in the potential are

$$V_F = |\lambda_H H_d H_u + \kappa S^2|^2 + \lambda_H^2 |S|^2 \left(|H_u|^2 + |H_d|^2\right), \tag{A.3}$$

which gives

$$\langle V_F \rangle = \lambda_H^2\, v_u^2\, v_d^2\, \cos^2\theta + 2\lambda_H\, \kappa\, x^2 v_u\, v_d\, \cos\theta \cos(2\gamma - \alpha) + \kappa^2 x^4 + \lambda_H^2 x^2 (v_u^2 + v_d^2)\,. \tag{A.4}$$

The relevant soft SUSY breaking terms are

$$V_{\text{soft}} = m_{Hu}^2 |H_u|^2 + m_{Hd}^2 |H_d|^2 + m_S^2 |S|^2 + \left(\lambda_H A_{\lambda H} S H_d H_u + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}\right). \tag{A.5}$$

The soft terms are all real because all the couplings are real, so this gives

$$\begin{aligned} \langle V_{\text{soft}} \rangle = {} & m_{Hu}^2\, v_u^2 + m_{Hd}^2\, v_d^2 + m_S^2\, x^2 + \\ & + 2\lambda_H A_{\lambda H}\, x\, v_u v_d \cos\theta \cos(\gamma + \alpha) + \frac{2}{3}\kappa\, A_\kappa x^3 \cos(3\gamma)\,. \end{aligned} \tag{A.6}$$

Finally, the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ D terms are

$$V_D = \frac{1}{8}(g_1^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g_2^2 |H_u^\dagger H_d|^2\,, \tag{A.7}$$

which gives

$$\langle V_D \rangle = \frac{1}{8}(g_1^2 + g_2^2)(v_u^2 - v_d^2)^2 + \frac{1}{2} g_2^2\, v_u^2 v_d^2\, \sin^2\theta\,. \tag{A.8}$$

The sum of eqs. (A.4), (A.6), and (A.8) is to be minimized with respect to v_u, v_d, and x. We have checked explicitly that all scalar mass-squared terms are positive at the solution, so we have at least a local minimum.

We are only interested in minima that preserve $\mathrm{U}(1)_{\mathrm{EM}}$ and do not spontaneously violate CP, so we will assume that $\theta = \alpha = \gamma = 0$ from now on.

We now turn to the fermion mass matrices. Anomaly mediation predicts the phase of the gaugino masses, and so we must be careful about relative signs. We

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define the anomaly-mediated gaugino masses to be given by eq. (1.3). The neutralinos $\{\tilde{B}, \tilde{W}_3, \tilde{H}_u^0, \tilde{H}_d^0, \tilde{S}\}$ have (Majorana) mass matrix

$$\mathcal{M} = \begin{pmatrix} -m_1 & 0 & g_1 v_u/\sqrt{2} & -g_1 v_d/\sqrt{2} & 0 \\ 0 & -m_2 & -g_2 v_u/\sqrt{2} & g_2 v_d/\sqrt{2} & 0 \\ g_1 v_u/\sqrt{2} & -g_2 v_u/\sqrt{2} & 0 & \lambda_H x & \lambda_H v_d \\ -g_1 v_d/\sqrt{2} & g_2 v_d/\sqrt{2} & \lambda_H x & 0 & \lambda_H v_u \\ 0 & 0 & \lambda_H v_d & \lambda_H v_u & 2\kappa x \end{pmatrix}. \tag{A.9}$$

In addition, there are neutral fermions $\{\tilde{R}_j, \tilde{H}'_{uj}, \tilde{H}'_{dj}\}$ with mass matrix

$$\mathcal{M}_{\text{neutralino},j} = \begin{pmatrix} b_j x & -a_{dj} v_d & a_{uj} v_u \\ -a_{dj} v_d & 0 & \lambda'_{Hj} x \\ a_{uj} v_u & \lambda'_{Hj} x & 0 \end{pmatrix}. \tag{A.10}$$

In the basis $\{\tilde{W}^+, \tilde{H}_u^+\}$, $\{\tilde{W}^-, \tilde{H}_d^-\}$ the chargino (Dirac) mass matrix is

$$\mathcal{M} = \begin{pmatrix} -m_2 & g_2 v_d \\ g_2 v_u & -\lambda_H x \end{pmatrix}. \tag{A.11}$$

We now consider the scalar mass matrices. Write

$$\begin{aligned} H_u &= \begin{pmatrix} H_u^+ \\ v_u + (h_u^0 + iA_u^0)/\sqrt{2} \end{pmatrix}, & H_d &= \begin{pmatrix} v_d + (h_d^0 + iA_d^0)/\sqrt{2} \\ H_d^- \end{pmatrix}, \\ S &= \frac{x + s + iA_S}{\sqrt{2}}, & R &= \frac{r + iA_R}{\sqrt{2}}, \end{aligned} \tag{A.12}$$

etc. The mass matrix for the CP-even neutral bosons $\{h_u^0, h_d^0, s\}$ is

$$\begin{aligned} (\mathcal{M}^2)_{11} &= m_{Hu}^2 + \lambda_H^2 (v_d^2 + x^2) + \frac{1}{4}(g_1^2 + g_2^2)(3v_u^2 - v_d)^2 , \\ (\mathcal{M}^2)_{12} &= 2\lambda_H^2 v_u v_d + \lambda_H \kappa x^2 + \lambda_H A_{\lambda H} x - \frac{1}{2}(g_1^2 + g_2^2) v_u v_d , \\ (\mathcal{M}^2)_{13} &= 2\lambda_H^2 x v_u + 2\lambda_H \kappa x v_d + \lambda_H A_{\lambda H} v_d , \\ (\mathcal{M}^2)_{22} &= m_{Hd}^2 + \lambda_H^2 (v_u^2 + x^2) + \frac{1}{4}(g_1^2 + g_2^2)(3v_d^2 - v_u^2) , \\ (\mathcal{M}^2)_{23} &= 2\lambda_H^2 x v_d + 2\lambda_H \kappa x v_u + \lambda_H A_{\lambda H} v_u , \\ (\mathcal{M}^2)_{33} &= m_S^2 + \lambda_H^2 (v_u^2 + v_d^2) + 2\lambda_H \kappa v_u v_d + 6\kappa^2 x^2 + 6\kappa A_\kappa x . \end{aligned} \tag{A.13}$$

The CP-even bosons $\{h'^0_{uj}, h'^0_{dj}, r_j\}$ have mass matrix

$$\begin{aligned} (\mathcal{M}_j^2)_{11} &= m_{H'uj}^2 + a_{dj}^2 v_d^2 + \lambda'^2_{Hj} x^2 + \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2) , \\ (\mathcal{M}_j^2)_{12} &= (\lambda_H \lambda'_{Hj} + a_{uj} a_{dj}) v_u v_d + \kappa \lambda'_{Hj} x^2 + \lambda'_{Hj} A'_{Hj} x , \\ (\mathcal{M}_j^2)_{13} &= (\lambda_H a_{dj} + \lambda'_{Hj} a_{uj}) x v_u + b_j a_{dj} x v_d + a_{dj} A_{adj} v_d , \end{aligned}$$

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$$
\begin{aligned}
(\mathcal{M}_j^2)_{22} &= m^2_{H'dj} + a^2_{uj} v^2_{uj} + \lambda'^2_{Hj} x^2 - \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)\,, \\
(\mathcal{M}_j^2)_{23} &= \lambda_H a_{uj} x v_d + b_j a_{uj} x v_u + \lambda'_{Hj} a_{dj} x v_d + a_{uj} A_{auj} v_u\,, \\
(\mathcal{M}_j^2)_{33} &= m^2_{Rj} + b_j^2 x^2 + b_j \kappa x^2 + b_j \lambda_H v_d v_u + a^2_{uj} v_u^2 + a^2_{dj} v_d^2 + 2 b_j A_{bj} x\,. \qquad \text{(A.14)}
\end{aligned}
$$

The CP-odd bosons $\{A_u^0, A_d^0, A_S\}$ have mass matrix

$$
\begin{aligned}
(\mathcal{M}^2)_{11} &= m^2_{Hu} + \lambda_H^2 (v_d^2 + x^2) + \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)\,, \\
(\mathcal{M}^2)_{12} &= -\lambda_H \kappa x^2 - \lambda_H A_{\lambda H} x\,, \\
(\mathcal{M}^2)_{13} &= 2\lambda_H \kappa x v_d - \lambda_H A_{\lambda H} v_d\,, \\
(\mathcal{M}^2)_{22} &= m^2_{Hd} + \lambda_H^2 (v_u^2 + x^2) - \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)\,, \\
(\mathcal{M}^2)_{23} &= 2\lambda_H \kappa x v_u - \lambda_H A_{\lambda H} v_u\,, \\
(\mathcal{M}^2)_{33} &= m_S^2 + \lambda_H^2 (v_u^2 + v_d^2) - 2\lambda_H \kappa v_u v_d + 2\kappa^2 x^2 - 6\kappa A_\kappa x\,. \qquad \text{(A.15)}
\end{aligned}
$$

This matrix has a zero eigenvalue corresponding to the Nambu-Goldstone boson absorbed in the Higgs mechanism. The CP-odd bosons $\{A'_{uj}, A'_{dj}, A_{Rj}\}$ have mass matrix

$$
\begin{aligned}
(\mathcal{M}_j^2)_{11} &= m^2_{H'uj} + a^2_{dj} v_d^2 + \lambda'^2_{Hj} x^2 + \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)\,, \\
(\mathcal{M}_j^2)_{12} &= (a_{uj} a_{dj} - \lambda_H \lambda'_{Hj}) v_u v_d - \kappa \lambda'_{Hj} x^2 - \lambda'_{Hj} A'_{Hj} x\,, \\
(\mathcal{M}_j^2)_{13} &= (\lambda'_{Hj} a_{uj} - \lambda_H a_{dj}) x v_u + b_j a_{dj} x v_d - a_{dj} A_{adj} v_d\,, \\
(\mathcal{M}_j^2)_{22} &= m^2_{H'dj} + a^2_{uj} v_u^2 + \lambda'^2_{Hj} x^2 - \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 - v_d^2)\,, \\
(\mathcal{M}_j^2)_{23} &= (\lambda'_{Hj} a_{dj} - \lambda_H a_{uj}) x v_d + b_j a_{uj} x v_u - a_{uj} A_{auj} v_u\,, \\
(\mathcal{M}_j^2)_{33} &= m^2_{Rj} + b_j^2 x^2 - b_j \kappa x^2 - \lambda_H b_j v_u v_d + a^2_{uj} v_u^2 + a^2_{dj} v_d^2 - 2 b_j A_{bj} x\,. \qquad \text{(A.16)}
\end{aligned}
$$

The gaugino masses are determined from the beta functions by

$$
m_\lambda = m_{3/2}\, \frac{\beta_g}{g}\,, \qquad \text{(A.17)}
$$

where $\beta_g = dg/d\ln\mu$. The 1-loop gauge beta functions are

$$
\begin{aligned}
16\pi^2 \beta_3 &= -(3 - n_3) g_3^3\,, \\
16\pi^2 \beta_2 &= (1 + n_2) g_2^3\,, \\
16\pi^2 \beta_1 &= \Big(11 + n_2 + \frac{2}{3} n_3\Big) g_1^3\,, \qquad \text{(A.18)}
\end{aligned}
$$

where n_3 is the number of pairs of vector-like triplets $(\mathbf{3}, \mathbf{1})_{-2/3} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{2/3}$, and n_2 is the number of pairs of vector-like doublets $(\mathbf{1}, \mathbf{2})_1 \oplus (\mathbf{1}, \mathbf{2})_{-1}$, relative to the MSSM.

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(In our model, $n_2 = 3$, $n_5 = 1$.) The numerical values of the gauge couplings in $\overline{\rm DR}$ are [25]

$$g_1(1\,\mathrm{TeV}) = 0.363\,, \qquad g_2(1\,\mathrm{TeV}) = 0.638\,, \qquad g_3(1\,\mathrm{TeV}) = 1.1\,. \tag{A.19}$$

The scalar mass-squared parameters can be easily read off from the anomalous dimensions

$$\gamma_a = \frac{d \ln Z_a}{d \ln \mu} \tag{A.20}$$

as

$$m_a^2 = -\frac{m_{3/2}^2}{4}\frac{d\gamma_a}{d\ln\mu} = -\frac{m_{3/2}^2}{4}\sum_g \frac{\partial\gamma_a}{\partial g}\beta_g\,, \tag{A.21}$$

where the sum runs over all couplings g, and $\beta_g = dg/d\ln\mu$. The A terms are given by

$$A_{abc} = \frac{m_{3/2}}{2}(\gamma_a + \gamma_b + \gamma_c)\,. \tag{A.22}$$

In the present model, the 1-loop anomalous dimensions are given by

$$\begin{aligned}
16\pi^2\gamma_S &= -4\lambda_H^2 - 6\lambda_T^2 - 4\kappa^2 - \sum_{j=1}^{3}\left(4\lambda_{Hj}^{\prime 2} + b_j^2\right), \\
16\pi^2\gamma_{Hu} &= -2\lambda_H^2 - 6y_t^2 + 3g_2^2 + g_1^2 - 2\sum_{j=1}^{3} a_{uj}^2\,, \\
16\pi^2\gamma_{Hd} &= -2\lambda_H^2 - 6y_b^2 - 2y_\tau^2 + 3g_2^2 + g_1^2 - 2\sum_{j=1}^{3} a_{dj}^2\,, \\
16\pi^2\gamma_{H'uj} &= -2\lambda_{Hj}^{\prime 2} + 3g_2^2 + g_1^2 - 2a_{dj}^2\,, \\
16\pi^2\gamma_{H'dj} &= -2\lambda_{Hj}^{\prime 2} - 2y_{ej}^{\prime 2} + 3g_2^2 + g_1^2 - 2a_{uj}^2\,, \\
16\pi^2\gamma_T &= -2\lambda_T^2 + \frac{16}{3}g_3^2 + \frac{4}{9}g_1^2\,, \\
16\pi^2\gamma_{Rj} &= -4a_{uj}^2 - 4a_{dj}^2 - 2b_j^2\,, \\
16\pi^2\gamma_{tL} &= -2y_t^2 - 2y_b^2 + \frac{16}{3}g_3^2 + 3g_2^2 + \frac{1}{9}g_1^2\,, \\
16\pi^2\gamma_{tR} &= -4y_t^2 + \frac{16}{3}g_3^2 + \frac{16}{9}g_1^2\,, \\
16\pi^2\gamma_{bR} &= -4y_b^2 + \frac{16}{3}g_3^2 + \frac{4}{9}g_1^2\,, \\
16\pi^2\gamma_{\tau L} &= -2y_\tau^2 - 2y_{e3}^{\prime 2} + 3g_2^2 + g_1^2\,, \\
16\pi^2\gamma_{\tau R} &= -4y_\tau^2 - 4y_{e3}^{\prime 2} + 4g_1^2\,.
\end{aligned} \tag{A.23}$$

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